Turn Performance — Altitude Effects

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In two previous articles (Nov/Dec 2000 pp. 38–39 and Mar/Apr 2001 pp. 28–29) we looked at steady level turns. In those articles we found that the radius of the turn depends only on the velocity (TAS). However, we also found that the ability to maintain the *level* turn at a particular velocity (TAS) was constrained by stall and power available. Here we want to briefly look at the effects of altitude.

From previous articles recall that the effective power-required curve in a steady level turn is

$$P_{r_{\text{turn}}} = \underbrace{\text{Constant}\,\sigma f}_{\text{parasite}} + \underbrace{\frac{\text{Konstant}}{\sigma}\left(\frac{W}{b}\right)^2 \frac{1}{(\cos\phi)^2}}_{\text{effective induced}} \tag{1}$$

where σ (sigma) represents the ratio of density at any altitude to the density at sea level, f is a measure of the parasite drag, W is weight, b is the wing span and Constant and Konstant are constants. The second term on the right contains the load factor $n = 1/\cos\phi$. Hence, the effective induced power required increases by the load factor *squared*.

Sigma, σ, which represents the altitude effect, decreases with increasing altitude. Thus, Equation (1) shows that the parasite drag decreases with increasing altitude while the effective induced drag increases. As shown in Figure 1, this has profound effects on the ability to sustain a level turn.

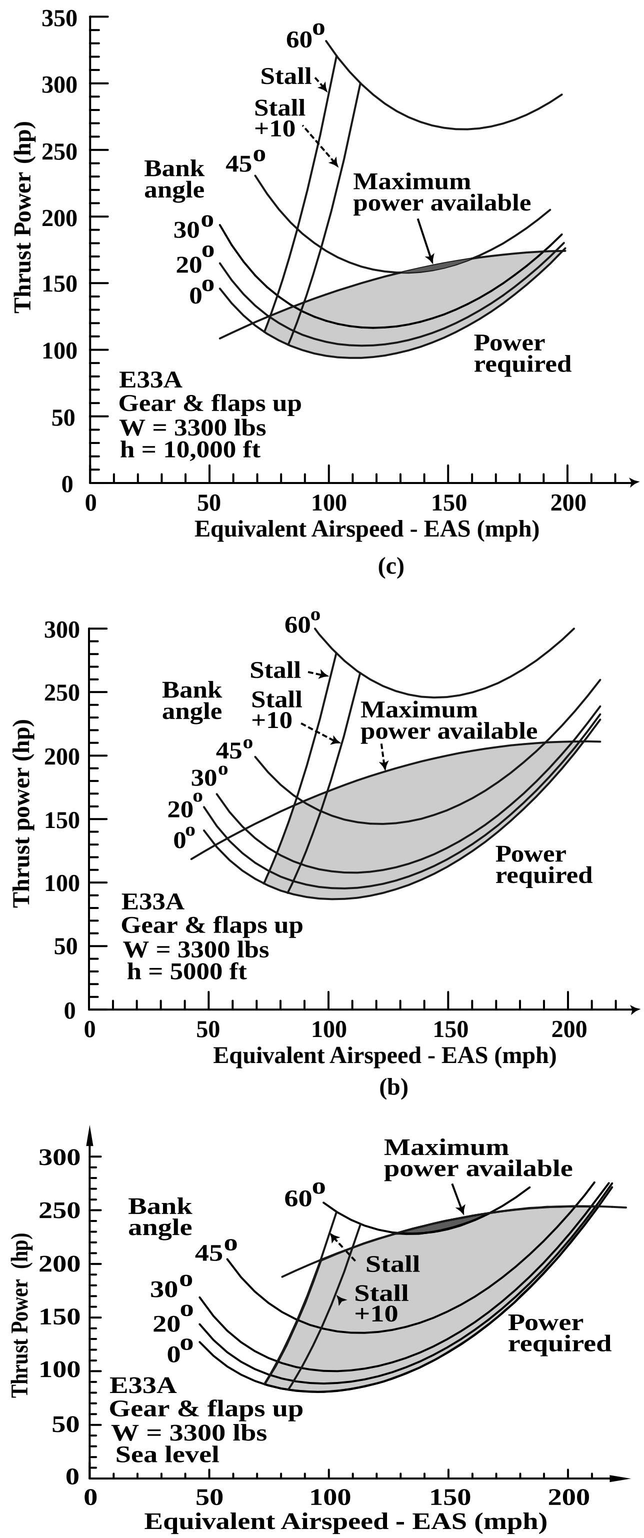

Figure 1. Power required and available in a steady level coordinated turn for various altitudes. (a) sea level; (b) 5,000 feet; (c) 10,000 feet.

Figure 1 shows the maximum power available and the banked power required for sea level, and for density altitudes of 5,000 and 10,000 feet. Here the curves are plotted as functions of equivalent airspeed (EAS) because, if we neglect the effects of compressibility and instrument error, the equivalent airspeed is the same as indicated airspeed. Indicated airspeed is what we, as pilots, are interested in. Of course, at sea level equivalent airspeed is the same as true airspeed.

Recall from the previous article that there are two limiting factors to the ability to maintain a steady level turn. At the higher airspeeds typically power available is the limiting factor; at lower airspeeds stall is the limiting factor, although in some cases power available becomes the limiting factor. Figure 1a clearly shows that at sea level for a 60° bank both the upper and lower limits are determined by power available. Notice that at sea level there is a small range of equivalent airspeeds where even for a 60° bank the power available is greater than the banked power required. Hence, a small positive rate-of-climb can be achieved. However, notice from Figure 1b that for a density altitude of 5000 feet a 60° level banked turn can not be maintained for any equivalent airspeed. In addition, Figure 1c shows that at a density altitude of 10,000 feet there is only a very small range of equivalent airspeeds where a 45° level banked turn is possible.

Looking closely at Figure 1c, notice that a 30° degree level turn is *not* possible at the stall airspeed but is just possible at the stall airspeed plus 10 mph. Also notice that a 20° level turn *is* possible at the stall airspeed. This brings up an interesting question. If you must execute a *level* turn, is it better to fly at a higher velocity and a larger bank angle or at the minimum velocity at which a level turn can be maintained and with a smaller bank angle? The answer lies in the radius of turn equation from the first article in this series

$$R = \frac{V^2}{g\tan\phi} = \frac{(TAS)^2}{g\tan\phi} = \Big(\frac{EAS}{\sigma}\Big)^2 \frac{1}{g\tan\phi} \qquad (2)$$

Notice that in Eq.(2) the EAS is squared while the tangent of the bank angle is not. What this means is that for bank angles less than 45° degrees it is nearly always better to fly at the lower velocity and bank angle. For example, at 10,000 feet at an EAS of 125 mph and a bank angle of 45° the turn radius is 1924 feet, i.e.

$$R_{45} = \Big(\frac{(125)(1.47)}{0.7385}\Big)^2 \frac{1}{(32.174)(1)} = 1924\,\text{ft}$$

while for an EAS of 85 mph and a 30° bank angle the turn radius is 1541 feet, i.e.

$$R_{30} = \Big(\frac{(85)(1.47)}{0.7385}\Big)^2 \frac{1}{(32.174)(0.5774)} = 1541\,\text{ft}$$

which is a 20% decrease in the turn radius.

Figure 2 represents the gray areas of the power available–power required curves in Figure 1 as well as those for altitudes of 15,000 and 17,500 feet. Recall that the outer boundaries of the gray areas represent the curves of EAS and bank angle for sustained level turns. The interior of the gray areas represent combinations of EAS and bank angle for which the aircraft can climb while turning.

Because the abcissa (horizontal axis) in Figure 2 is EAS, the stall curve is the same for all altitudes. We can see this by recalling that $\sqrt{\sigma}$ TAS = EAS and

$$\sqrt{\sigma}\, V_{\text{stall}_\phi} = \sqrt{\sigma}\,\text{TAS}_{\text{stall}_\phi} = \text{EAS}_{\text{stall}_\phi}$$

$$= \sqrt{\frac{2W/S}{\rho_{\text{SL}} C_{L_{\max}}} \frac{1}{\cos\phi}} = V_{\text{stall}_\phi} \text{ at sea level}$$

From Figure 2, as expected, we also notice that as the altitude increases the area under the curves decreases and shifts to the left, indicating a smaller range of EASs where the aircraft can maintain a level turn or exercise a climbing turn for a given bank angle. Notice also that, except for 17,500 feet, at some bank angle the aircraft can maintain a level coordinated turn at full throttle and 2700 RPM. This bank angle may be quite small, e.g., about 10° at 15,000 feet at the stall airspeed.

The curve for 17,500 feet illustrates that the aircraft cannot maintain level flight even with a zero bank angle at the stall velocity. As the altitude increases the curve continues to shrink, until at the absolute ceiling it becomes a point and unbanked level flight is only possible at one speed. At the absolute altitude any turn results in a descent.

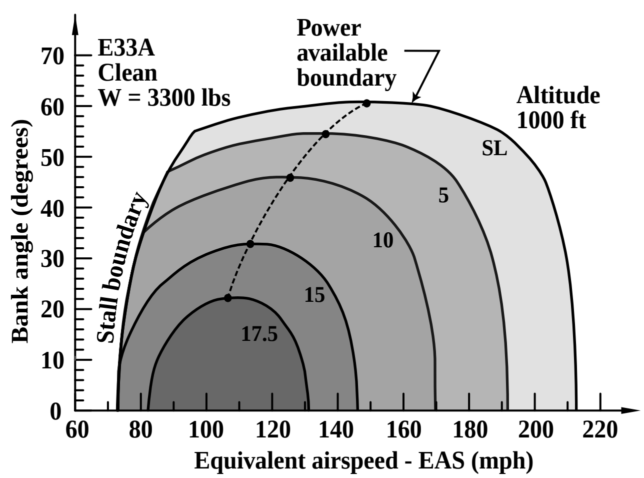

Figure 2. Possible bank angles in a steady level coordinated turn for various altitudes.

Dave Rogers' previous articles are available through a link at his website at *www.nar-associates.com*.